# —PLANE TRUSSES—

*. Truss is a Rigid structure in which members are subjected to either axial tensile or axial compressive load only.

*. Bending moment is zero everywhere in the structure.

*. The external force on the structure, the internal force in the bars and the reactions at the supports all are taken to act in one and same plane, therefore this is termed as plane trusses.

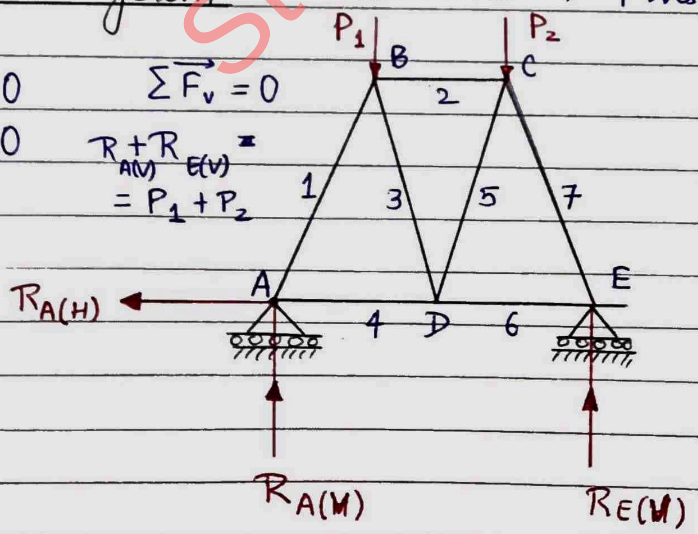
⇒ Assumptions for Truss :-

1. All members should be pin jointed or hinged only.
2. Only concentrated point-loads should be applied.
3. The self weight of the members are neglected.
4. The structure is load only at the joint.

⇒ Truss System = members + Pins / joints

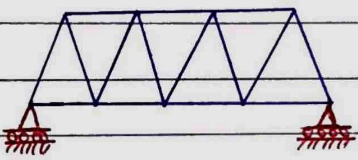
$\Sigma \vec{F}_H = 0$ $\qquad$ $\Sigma \vec{F}_v = 0$

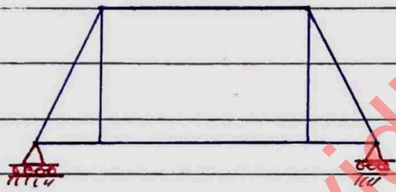
$R_{A(H)} = 0$ $\qquad$ $R_{A(V)} + R_{E(V)} = P_1 + P_2$

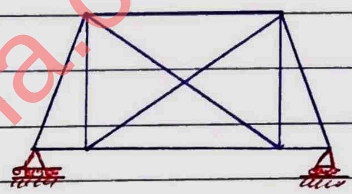


## ⇒ Types of Plane Trusses [2-D]

| Stable/perfect/determinate/efficient | Unstable/imperfect/collapsible/deficient | redundant/indestructible/indeterminant |
| --- | --- | --- |
| if $\boxed{m = 2j - 3}$ | if $\boxed{m < 2j - 3}$ | if $\boxed{m > 2j - 3}$ |







NOTE :- ① members under tension are called 'tie'.
② member under compression are called 'strut'.
③ $m$ = no. of members
④ $j$ = no. of joint
⑤ degree of freedom of truss is zero.

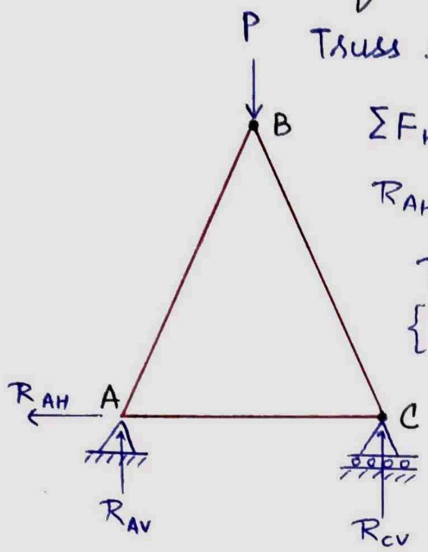
⇒ $\boxed{m + r = 2j}$

$m$ → no. of member
$r$ → no. of independent reactions
$j$ → no. of joints

## ⇒ Interaction of Loads, Reaction & Internal Forces

Truss system = members + Pins

Internal forces

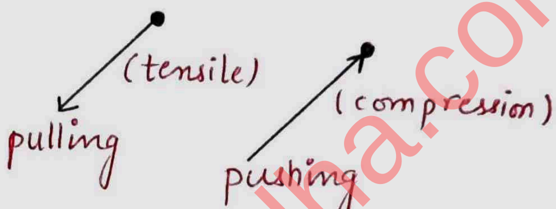


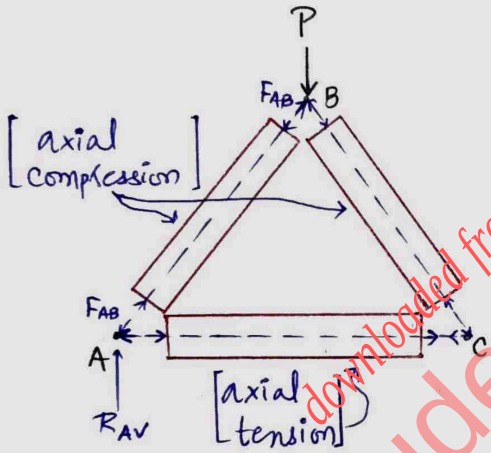

$\Sigma F_H = 0$

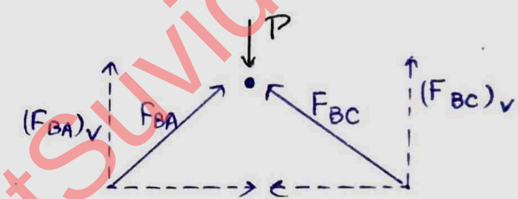
$R_{AH} = 0$

$\Sigma F_V = 0$

$R_{AV} + R_{CV} = P$
{vertically upward & non-zero}




Correct assumption

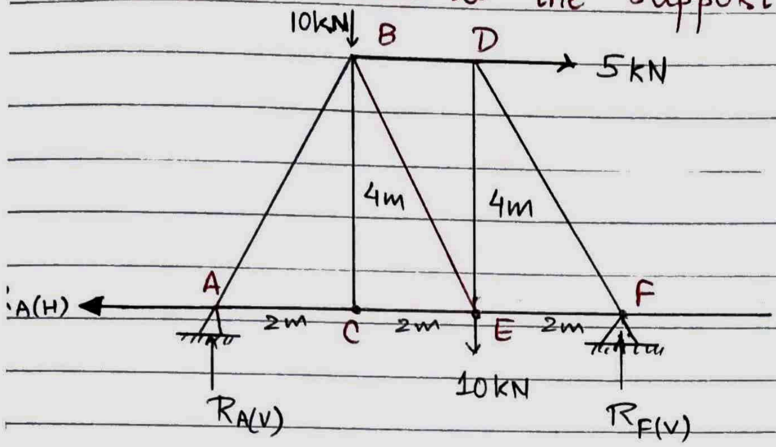
* vertical force components will cancel out the load
* horizontal force components will cancel out each other.

→ During analysis of a truss, we find compression/tension in members and their magnitude.

→ If a member pulls a joint, then certainly it is going to pull another joint with same magnitude and vice-versa for compression.

→ If any member has bending moment, then it is not considered as Truss because Bending Moment is zero in plane truss.

1 - Find reactions at the supports.




Sol:
$\Sigma \vec{F}_H = 0 \Rightarrow 5 - R_{A(H)} = 0 \Rightarrow \boxed{R_{A(H)} = 5 \text{ kN}}$

$\Sigma \vec{F}_V = 0 \Rightarrow R_{F(V)} + R_{A(V)} - 10 - 10 = 0$

$\Rightarrow \boxed{R_{F(V)} + R_{A(V)} = 20}$

Taking moment about A, $\Sigma M_A = 0$

$\Rightarrow 10 \times 2 + 10 \times 4 + 5 \times 4 - R_{F(V)} \times 6 = 0$

$\Rightarrow \boxed{R_{F(V)} = 13.33 \text{ kN}}$

$\Rightarrow \boxed{R_{A(V)} = 6.67 \text{ kN}}$

⇒ If two non-collinear members meet at an unloaded joint, both are zero force member.

⇒ If three members meet in an unloaded joint of which two member are collinear then the third member is a zero force member.

**Concept** :- Equilibrium of a joint is considered to find out loading in truss members.

⇒ **Method of Joint** :- This is based upon the equilibrium system.

↳ **Procedure** → ① find reactions at supports

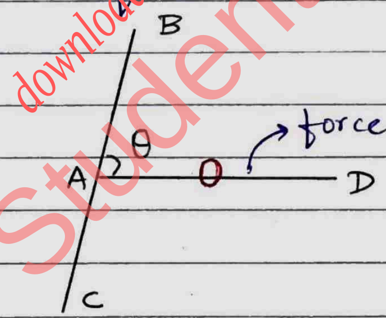
② consider the equilibrium of a joint where only 2 unknown members are meeting and then use –

$\Sigma \vec{F_x} = \Sigma \vec{F_y} = 0$ to find the unknowns.

☞ NOTE :- ① If a member pulls a joint then the member itself will be in tension with the same intensity

② If a member pushes a joint then the member itself will be in compression with same intensity.

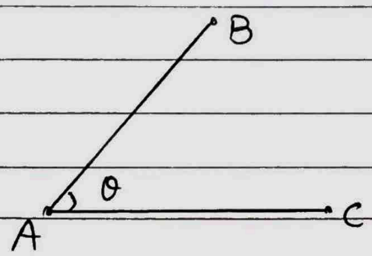
③ If at a joint 3 members are meeting and 2 are collinear then the force in the IIIrd member will be zero (if there is no load/reaction at that joint).




force in this member is zero

$\vec{F_{AD}} = 0$

④ If at a joint 2 members are meeting and they are non-collinear then force in both the member are zero (there is no load / reaction at that joint)

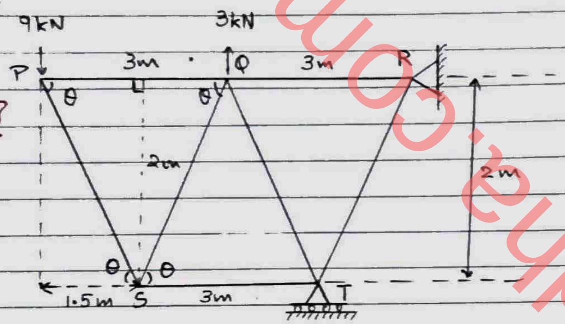



$\vec{F_{AB}} = 0$

$\vec{F_{AC}} = 0$

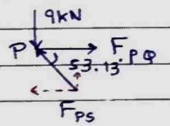
Problem 6 :-

find $F_{QS} = ?$




Sol$^n$.

Joint P → $\tan\theta = \frac{2}{1.5} \Rightarrow \theta = 53.13°$

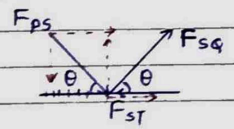



By Lami's theorem ← no need

Vertical component of $F_{PS}$ cancel 9kN load. ⇒ $F_{PS} \sin\theta = 9$

$F_{PS} = \frac{9}{\sin 53.13} = 11.25$ kN (comp.)

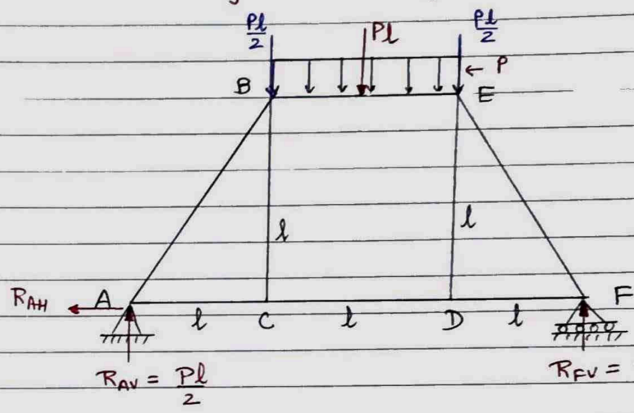
Joint S →




$F_{SQ} \sin\theta = F_{PS} \sin\theta$

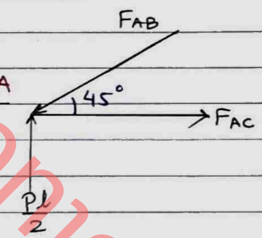
⇒ $F_{SQ} = 11.25$ kN (tensile)

Problem 7 :- find $F_{CD} = ?$




Sol$^n$.

Joint A




$F_{AB} \sin 45° = \frac{Pl}{2}$ $\{\because \Sigma F_Y = 0\}$ (1)

$F_{AB} \cos 45° = F_{AC}$ $\{\because \Sigma F_X = 0\}$ (2)

→ $F_{AC} = \frac{Pl}{2}$ (tensile)

Joint C


$F_{CD} = \frac{Pl}{2}$ (tensile)

⇒ Method of Section :- This method is convenient when the forces in a few members of a frame are required to be found out.

↳ Procedure → ① find reactions at support

② Cut the member under consideration by a section 1---1 and consider eq$^n$. of either left hand side or RHS of section 1---1 for applied loads, reactions and forces in the cut members and use ↴

$$\boxed{\begin{matrix}\Sigma \vec{F}_x = \Sigma \vec{F}_y = 0 \\ \text{and} \quad \Sigma \vec{M} = 0\end{matrix}}$$

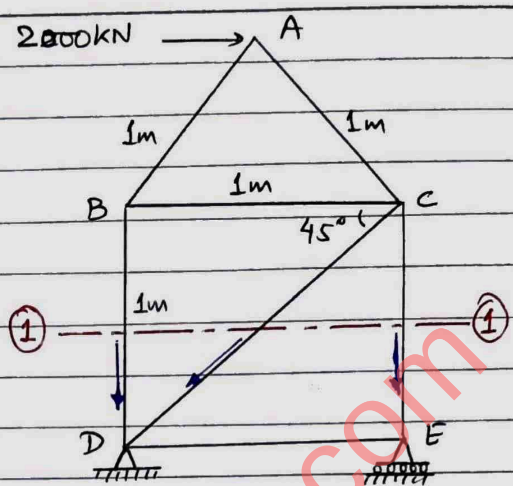
☞ NOTE :- ① Advantage of method of section is that, force in any intermediate member can be found directly without finding force in any other member.

② Cut the members such that entire truss is divided into two separate parts

③ Preferably do not cut more than 3 members. because in method of section we have only 3 equations of equilibrium

④ Cut the member such that all the cut members do not meet at same joint (if they meet at same joints it becomes of method of joint only).

Problem 2 → find $F_{DC} = ?$
[ISRO 2015]



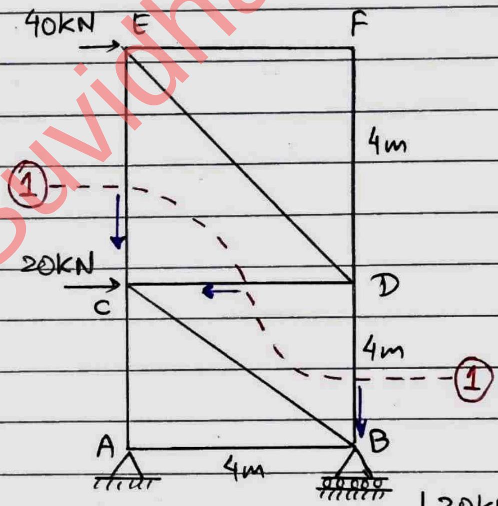

Solⁿ. Considering upper side of
1 --- 1

$\Sigma \vec{F}_H = 0$

$\Rightarrow 2000 - F_{DC} \cos 45° = 0$

$F_{DC} = 2828.42$ (tensile)

Problem 3 → find $F_{CD} = ?$



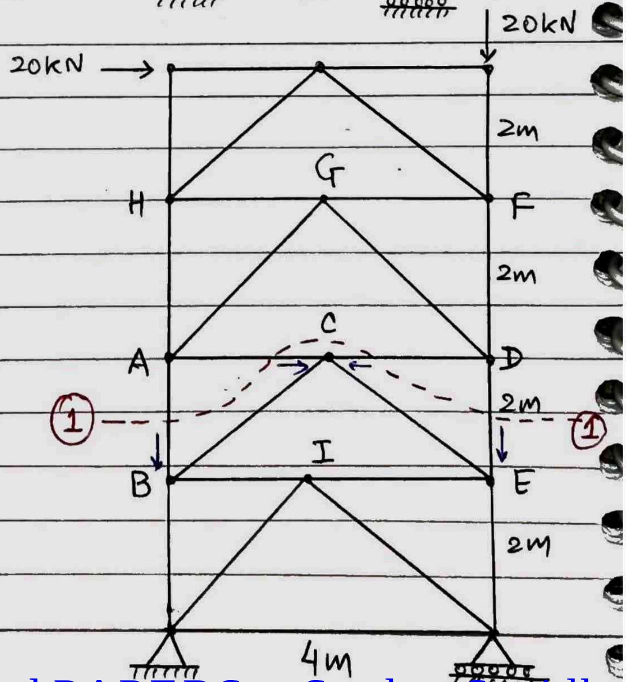

Solⁿ. Considering upper side of
1 --- 1

$\Sigma \vec{F}_H = 0$

$\Rightarrow 40 - F_{CD} = 0$

$F_{CD} = 40kN$ (tensile)

Problem 4 → find $F_{AB} = ?$




Solⁿ. Considering upper side of
1 --- 1

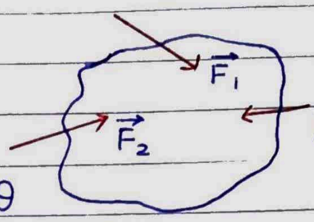
$\Sigma M_D = 0$

$20 \times 4 - F_{AB} \times 4 = 0$

$F_{AB} = 20KN$ (tensile)

# VIRTUAL WORK

⇒ Workdone :-




$W_{\vec{F_1}} = \vec{F_1} \cdot \vec{ds}$ ↙ independent

$$\text{Workdone} = \vec{F} \cdot \vec{ds} = F . ds . \cos\theta$$
$$= T . d\theta$$

$$\boxed{\begin{aligned} \text{Power} &= \frac{F . ds}{dt} = \vec{F} \cdot \vec{v} \\ &= \frac{T d\theta}{dt} = T\omega \\ &= \frac{2\pi N T}{60} \end{aligned}}$$

$W_{F_1} = \vec{F_1} \cdot \vec{ds}$    $\theta \ll$   $T \rightarrow +ve$

$W_{F_2} = \vec{F_2} \cdot \vec{ds}$    $\theta \gg$   $T \rightarrow -ve$

$W_{F_3} = \vec{F_3} \cdot \vec{ds}$

| |
|---|
| $ds \rightarrow$ actual displacement |
| $\partial s \rightarrow$ virtual displacement |

$$\text{Workdone, } W_{Total} = W_{F_1} + W_{F_2} + W_{F_3}$$
$$= (\vec{F_1} + \vec{F_2} + \vec{F_3}) \cdot \vec{ds}$$

$$\boxed{W_{Total} = \vec{F}_R \cdot \vec{ds}}$$

If body is in equilibrium
$\vec{F_R} = 0 \Rightarrow$ Total workdone $= 0$

⇒ Principle of Virtual work :-

It states that "if a system is in equilibrium then the sum of virtual workdone by all the forces will be zero".

Virtual Work = Force (virtual displacement)

$$\boxed{\Sigma (V.W.)_{ext.\ forces} = 0} \rightarrow \text{POVW}$$

☞ NOTE :- ① Using POVW, we can find forces which are necessary to keep a system in equilibrium.

② If using eqⁿ of equilibrium is difficult and time taken, then use POVW to find the unknown easily.

③ POVW is used in the system when no. of interconnected rigid bodies have degree of freedom more than zero but are in equilibrium.

④ In POVW, we do not need to find the reaction at supports because virtual work of reaction is always zero.

→ Procedure :-

(a) Take any fixed point in problem as origin, fix coordinate axis and find the coordinates of all the points where forces are acting.

(b) Find virtual displacements

(c) Use POVW to find the unknowns.

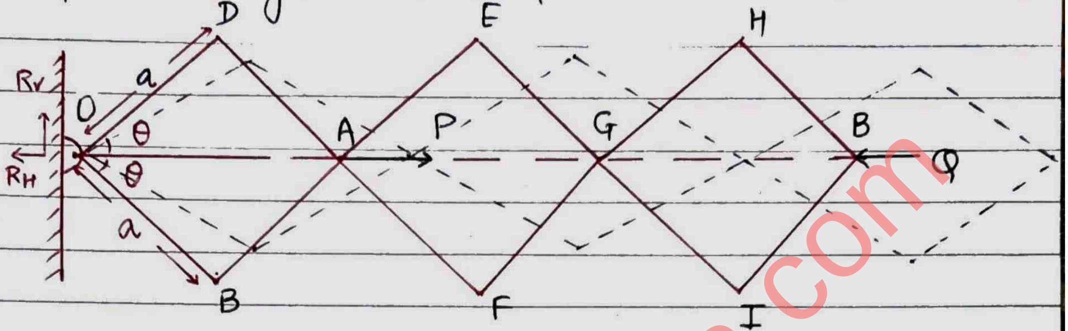
→ Sign Convention :-

1. Sign convention for the coordinates is choosen based on the quadrant in which they are lying.

2. If any force is acting along +ve x-axis or +ve y-axis then take that force as +ve and vice-versa.

Problem - 1 : For the lazy tong mechanism shown in figure, the relationship between P & Q so as to keep the system in equilibrium.

(a) $P = Q$
(b) $P = 3Q$
(c) $P = Q/2$
(d) $P = Q/3$




<u>Solⁿ</u>: There are three external forces $R_H$, P & Q.

<u>Method I</u> :- Assuming virtual displacement of system

$$(\partial s)_O = 0 \Rightarrow (V.W)_{R_H} = 0$$

Using POVW - $\Sigma V.W = 0$

$$P.(\partial s)_A + (-Q.(\partial s)_B) = 0$$

$$\Rightarrow P(x) - Q(3x) = 0$$

$$\Rightarrow \boxed{P = 3Q}$$

<u>Method II</u> :- Using coordinate system

1. $x_A = 2a\cos\theta \qquad x_B = 6a\cos\theta$

2. $\partial x_A = -2a\sin\theta.\partial\theta \qquad \partial x_B = -6a\sin\theta.\partial\theta$

3. $(V.W)_P + (V.W)_Q = 0$

$$\Rightarrow P(\partial x)_A + (-Q)(\partial x)_B = 0$$

$$\Rightarrow P(-2a\cancel{\sin\theta}\,\cancel{\partial\theta}) = -Q(6a\cancel{\sin\theta}.\cancel{\partial\theta})$$

$$\Rightarrow \boxed{P = 3Q}$$

Note ☞ If force is in horizontal, then only use x-coordinate. No need to use y-coordinate and also ... forces vertical ... take y-axis

**Friction**: when a body slides over another body, a force is exerted at the surface of contact by the stationary body on the moving body. This resisting force is called the force of friction and acts in a direction opposite to the direction of motion.

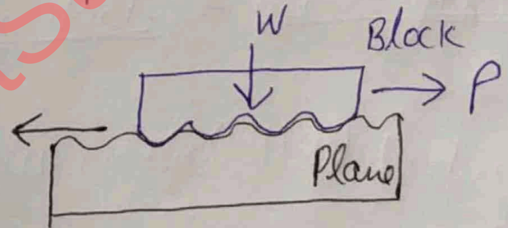
Friction is desirable as well as undesirable (a necessary evil)

Undesirable - Power screws
bearings and gear
flow of fluids in pipes

Desirable: Friction brakes and clutches
belt and rope drives
holding and fastening devices

Dry friction - The friction between dry surfaces in contact is called dry friction. It is also known as coulomb friction.
Types (Sliding friction & Rolling friction)

The major cause of such friction is due to minute projection (Irregularities) of the surfaces.



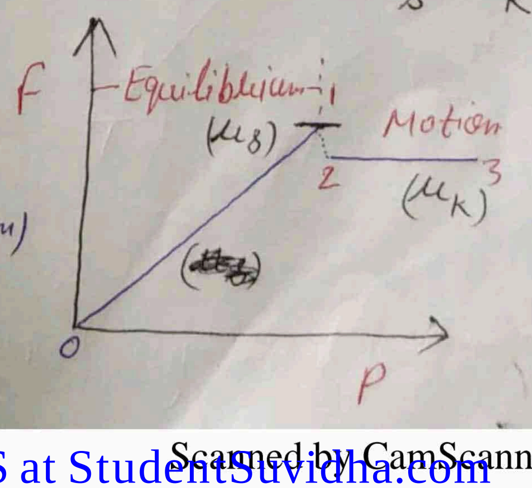

Fluid Friction -

Static and Dynamic friction -

Limiting friction - It is the maximum frictional force exerted at the time of impending motion.

$\mu_s > \mu_k$

0-1 = Zone of static friction.
2-3 - Zone of kinetic friction.
at point 1, friction is max. (Limiting friction)
1-2 - Variation is uncertain (dotted line)



## Laws of dry friction

1) The ~~total~~ (Limiting) friction that can be developed is independent of the magnitude of the area of contact.

2) The ~~total~~ Limiting friction that can be developed is proportional to the normal force transmitted across surface of contact.

3) The force necessary to start the motion is greater than that necessary to maintain the motion.

## Characterstics of friction

→ Friction always opposes the relative motion of the body and is tangential to the surface of contact.

→ It is a passive force, it exists as long as the tractive force acts.

→ It is a self-adjusting force

→ It is proportional to normal force.

# FRICTION

## Dry Friction / Coulomb Friction

↳ Whenever a body moves or tends to move tangentially wrt the surface on which it rest then, there is a oppositing force, which acts in the opposite direction of the movement of the body. This opposing force is called friction or force of friction

Friction
- 1. Static Friction ($f_s$)
- 2. Dynamic Friction ($f_k$)
  - (a) Sliding friction
  - (b) Rolling friction

1. Static Friction → The friction experienced by a body when it is at rest and has tendency of motion.

2. Dynamic Friction → The friction experienced by a body when it is in motion.
It is also called 'kinetic friction'.
(a) Sliding friction → The friction, experienced by a body when it slides over another body
(b) Rolling friction → The friction, experienced by a body when it rolls over another body

$0 < f_s \le (f_s)_{max}$

$(f_s)_{max} \propto N$

$\boxed{(f_s)_{max} = \mu_s . N}$

where $\mu_s$ = coeff. of static friction

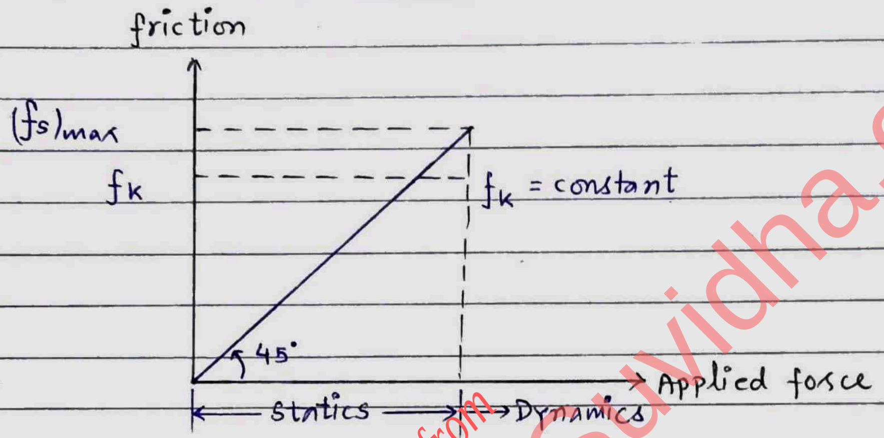
$f_k$ = constant = $\mu_k . N$

where $\mu_k$ = coeff. of kinetic friction

$\boxed{(f_s)_{max} > f_k}$

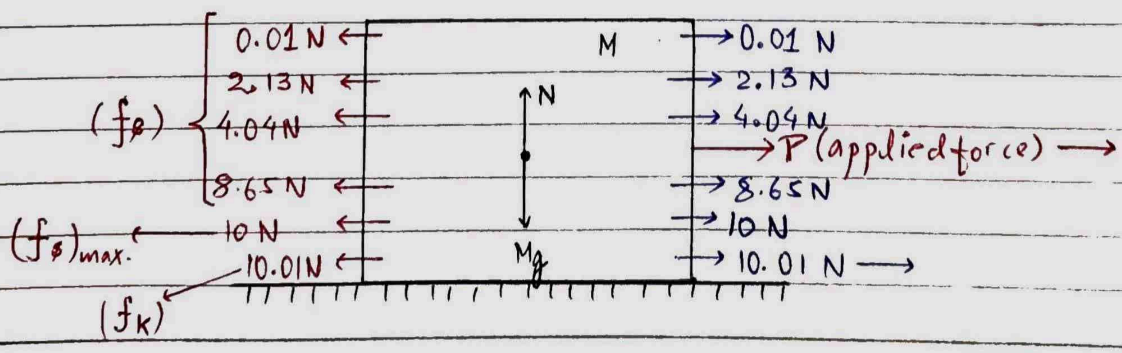
$\boxed{\mu_s > \mu_k}$

*. **Static friction** :- When two bodies in contact trying to slide w.r.t each others but they are not sliding then there exist static friction whose values varies from 0 to a maximum value $(f_s)_{max}$ depending on the value of applied force.




NOTE :- If the value of applied force is less than or equal to $(f_s)_{max}$, then there exist static friction whose value will be exactly equal to the applied load & will act opposite to the applied force.

*. $(f_s)_{max}$ is also called limiting friction because this is the maximum value of friction regardless of its type i.e. kinetic OR static.



## Terms related to friction

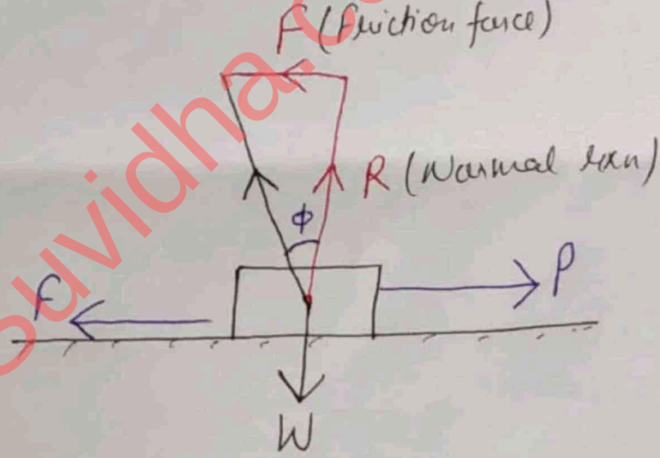
**Co-efficient of friction:** Ratio of force of friction to the normal reaction b/w the contact surfaces.

$$F \propto R$$

(Frictional force) (Normal reaction)

$$F = \mu R \qquad \mu_S = \frac{F_S}{R}, \quad \mu_K = \frac{F_K}{R}$$

**Angle of friction** - It is the angle which the resultant of normal reaction and limiting force of friction makes with the normal reaction.

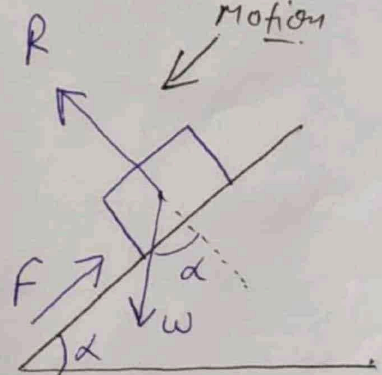



$$\tan\phi = \frac{F}{R}$$

$$\boxed{\mu = \tan\phi}$$

**Angle of repose:**

The angle of the inclined plane at which a block resting on it is about to slide down the plane is called the angle of repose. ($\alpha$)




$$R = W\cos\alpha \quad -(i)$$

$$\mu R = W\sin\alpha \quad -(ii)$$

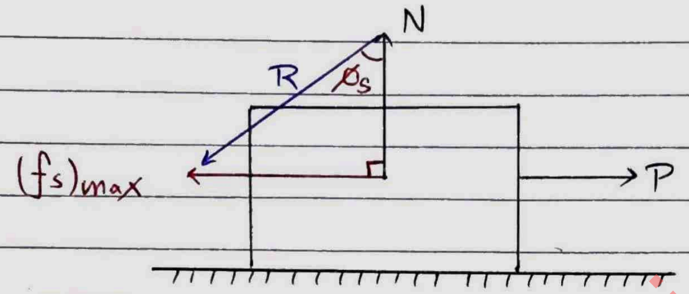
$$\mu = \tan\alpha$$

$$\tan\phi = \tan\alpha$$

$$\boxed{\phi = \alpha}$$

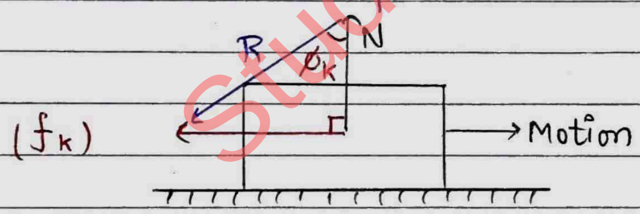
⇒ Angle of Static Friction, ($\phi_s$) →

It is the angle made by contact force with normal reaction when the body is at verge of motion.




⇒ Angle of Kinetic Friction, ($\phi_k$) →

It is the angle made by contact force with normal reaction when the body is in motion.




NOTE :- If $\mu_s$ & $\mu_k$ are not given separately

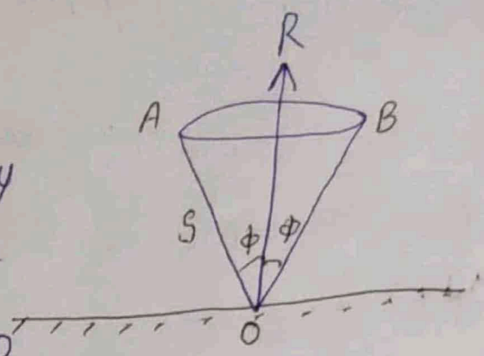
(i) $\mu_s = \mu_k = \mu$ (given)

(ii) $0 < f_s \leq (f_s)_{max} = \mu N = f_k$

(iii) $\tan \phi = \mu$

$\phi$ ↓ angle of friction

## Cone of friction:




The cone of friction is the imaginary cone AOB generated by revolving the static resultant about the normal OR.

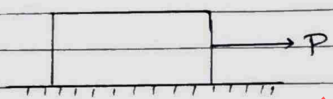
For the motion to occur the resultant R will lie on the surface of the cone.

when the friction force is less than the limiting friction, the total reaction would lies within the cone.

(This aspect forms the working principle for self-locking mechanisms.)

**Problem 1 :-** find the frictional force developed in the system shown below for P = 30, 50 & 60 N.

W = 100 Nt
$\mu_s$ = 0.5
$\mu_k$ = 0.4




**Tip :-** Always find values of $(f_s)_{max}$ & $(f_k)$ whether it is asked or not.

**Sol$^n$.**

N = 100 Newton → NFL

$(f_s)_{max.} = \mu_s N = 50$ Nt

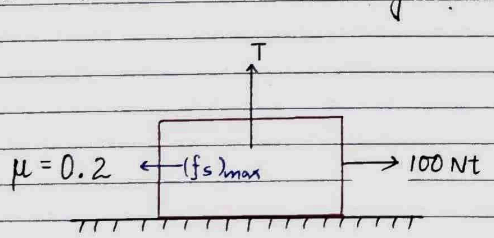
$f_k = \mu_k N = 40$ Nt

(i) if P = 30 Nt, $f = f_s = 30$ Nt (rest)

(ii) if P = 50 Nt
$f = (f_s)_{max} = 50$ Nt

(iii) if P = 60 Nt
$f = f_k = 40$ Nt (motion)

**Problem 2 :-** A block weighing 981 Nt is resting on a horizontal surface. The coeff. of friction b/w the block & the horizontal surface is 0.2. A vertical cable attached to the block provide partial support as shown. A man can pull horizontally with a force of 100 Nt. What will be tension (T) in the cable if the man is just able to move the block to the right?




**Sol$^n$:** According to question –

$(f_s)_{max} = 100$ Nt

0.2 N = 100 Nt

$\Rightarrow N = \frac{100}{0.2} = 500$ Nt

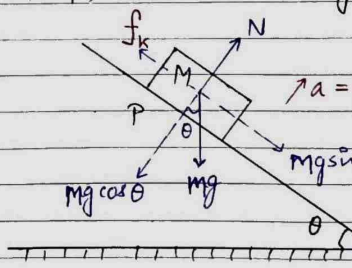
$\Rightarrow$ T + N = weight of the block =

T = 981 – 500 = 481 N

**Ques. 3.** A block of mass M is r
point P on a rough incli
inclination angle θ, shown
below. The coeff. of friction
μ < tan θ, then the time
block to ~~&~~ reach other poin
inclined plane where PQ = S,

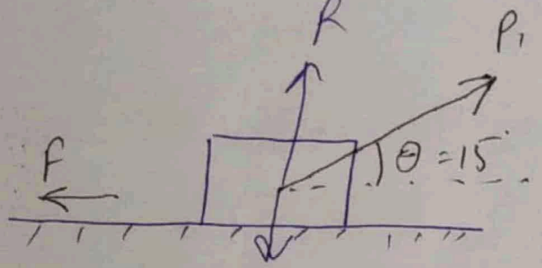
(a) $\sqrt{\frac{2S}{g(\cos\theta).(\tan\theta-\mu)}}$ (b)

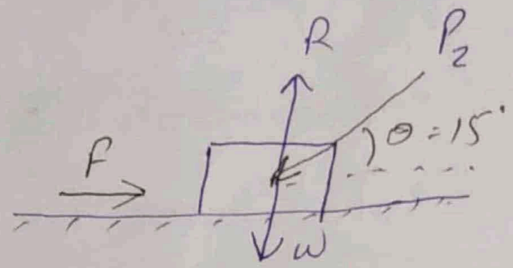
(c) $\sqrt{\frac{2S}{g\sin\theta(\tan\theta-\mu)}}$ (d)



Q A wooden block of weight 50N rests on a horizontal plane. Determine the force required to just (a) pull it (b) push it $\mu = 0.4$. Comment on the result.

Sol.




$\Sigma F_x = 0 \quad F = P_1 \cos 15$

$\Sigma F_y = 0 \quad R = W - P_1 \sin\theta$

$P_1 = 18.7\ N$

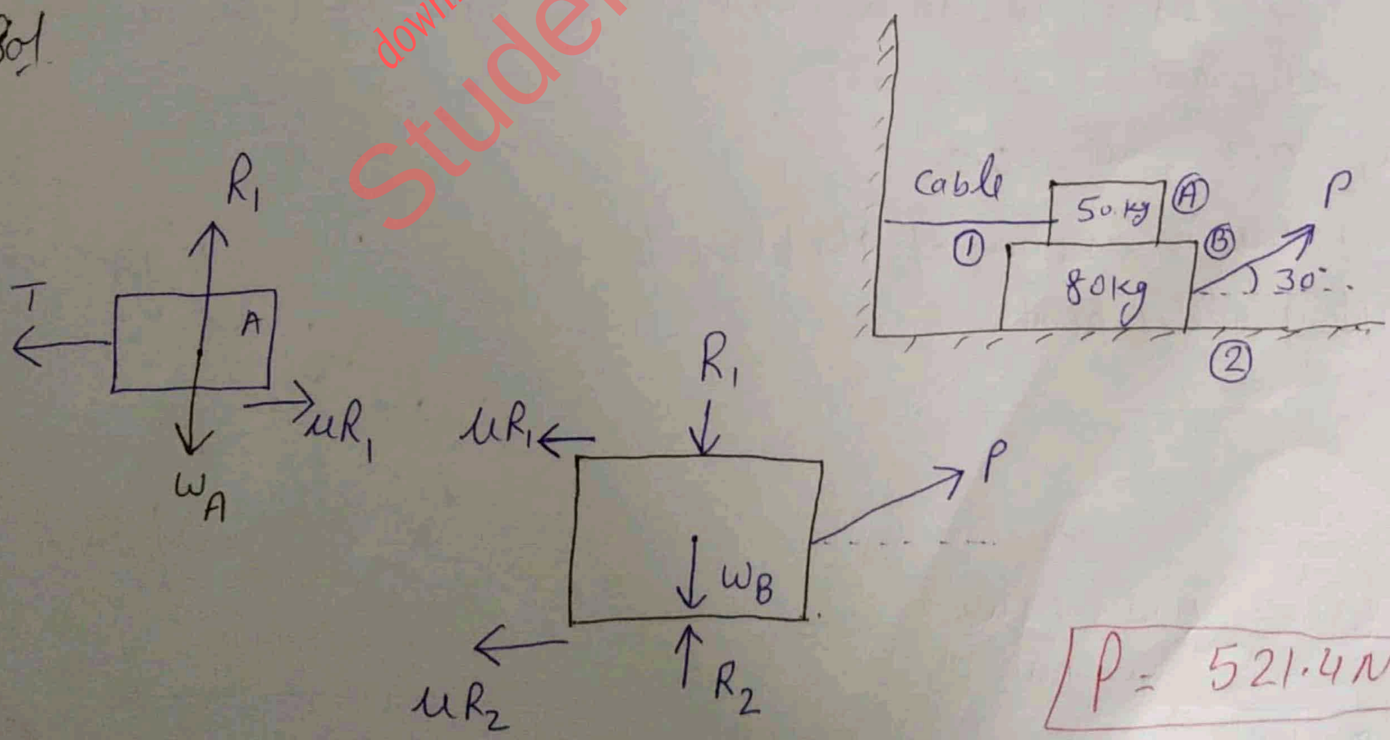
$P = 23.17\ N$

It is easier to pull the block than push it

---

Q Two blocks A and B weighing 50 kg and 80 kg resp are in equilibrium as shown in fig. Calculate the force P required to move the lower block B and tension in the cable. ($\mu = 0.3$)

Sol




$\boxed{P = 521.4N}$

Q A 7m long ladder rests against a vertical wall, with which it makes an angle of 45° with the floor. If a man, whose weight is one half of that of ladder climbs it, at what distance along the ladder will he be, when the ladder is about to slip?

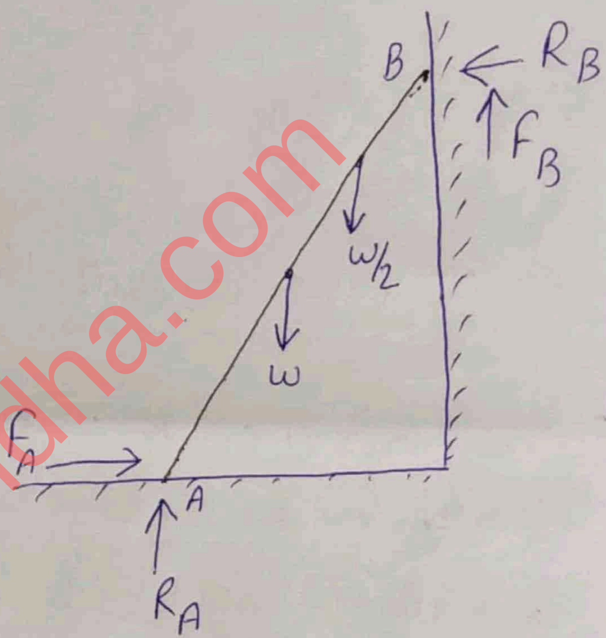
The Coefficients of friction b/w the ladder and wall is 1/3 and that between the ladder and the floor is 1/2.

Ans

$\sum F_x = 0$

$\sum F_y = 0$

$\sum M = 0$

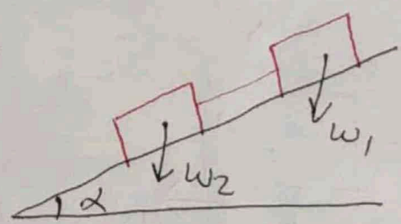



$\underline{\underline{l = 5m}}$

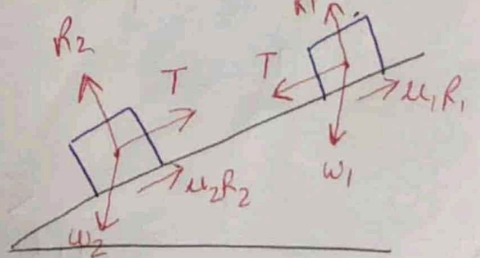
Q Two blocks of weigh $W_1 = 50N$ and $W_2 = 50N$ rest on a rough inclined plane as shown in fig.

$\mu_1 = 0.3$, $\mu_2 = 0.2$

Find the inclination of the plane for which slipping will impend.




Sol.

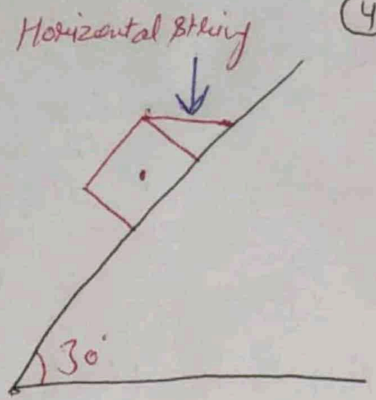



$$\tan\alpha = \frac{\mu_1 W_1 + \mu_2 W_2}{W_1 + W_2}$$

$\boxed{\alpha = 14°}$

Q. The block is tied up by a horizontal string which has a tension of 10N. If the block weighs 35 N, determine

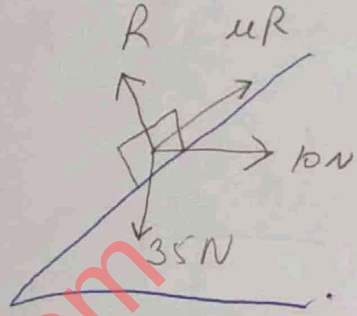
i) friction force on the block
ii) normal rxn of the inclined plane
iii) $\mu$.



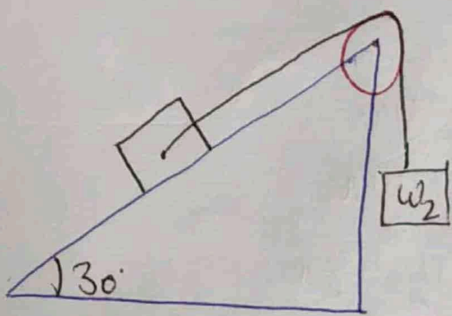

Sol: $f = \mu R = 8.84N$

$R = 35.31N$

$\mu = 0.25$

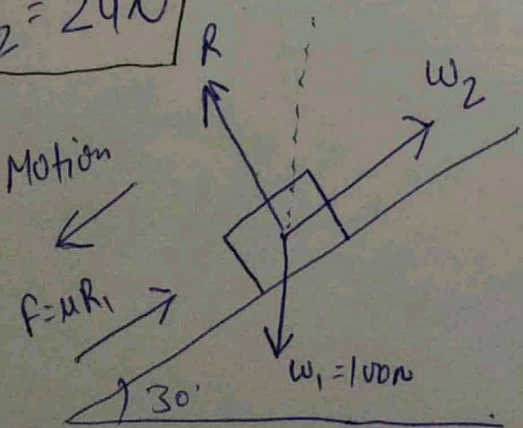



Q. A block of weight $W_1 = 100N$ rests on an inclined plane and another weight $W_2$ is attached to the first weight through a string as shown at fig no. If the $\mu$ b/w the block and plane is 0.3, determine the max. and min values of $W_2$ so that eq. can exist.

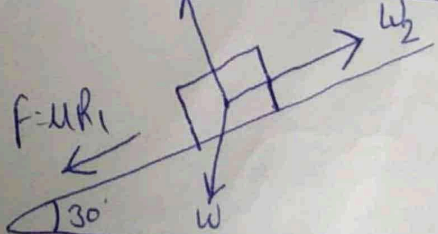



Sol.

$\boxed{W_2 = 24N}$



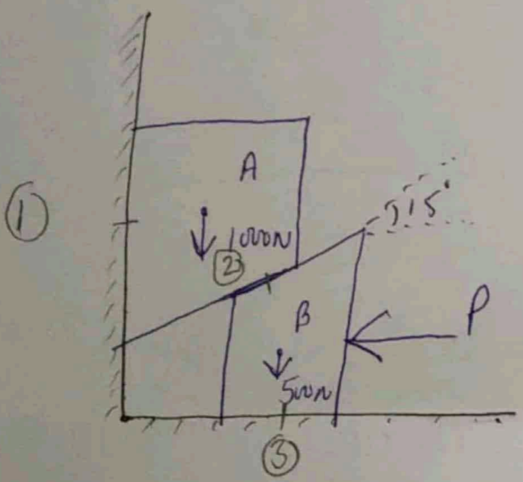

$\boxed{W_2 = 76N}$

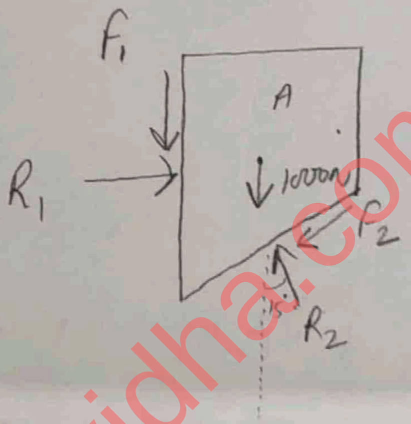


Q (wedge Problem) 6.9

a) Block A weighing 1000 N is to be raised by means ~~by~~ of a 15° wedge B weighing 500N. Assuming $\mu = 0.2$ for all contact. determine what minimum horizontal force P should be applied to raise the block.

b) Assuming that there is no friction between the block A and the vertical surface and wedge is of negligible weight, what is the minimum value of '$\mu$' required for the wedge to be self-locking?



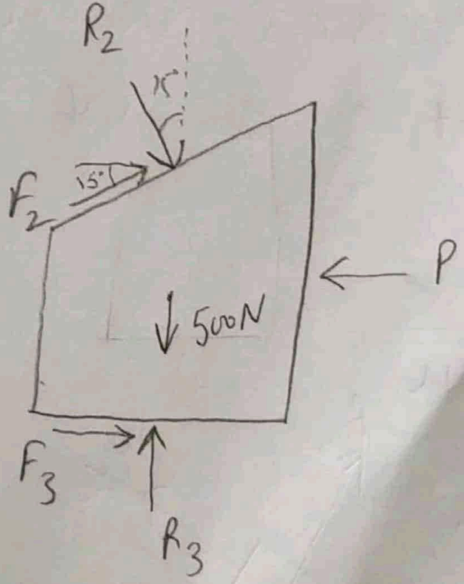

when upper block moving up, the lower block moving right to left.

$$\boxed{P = 871 N}$$

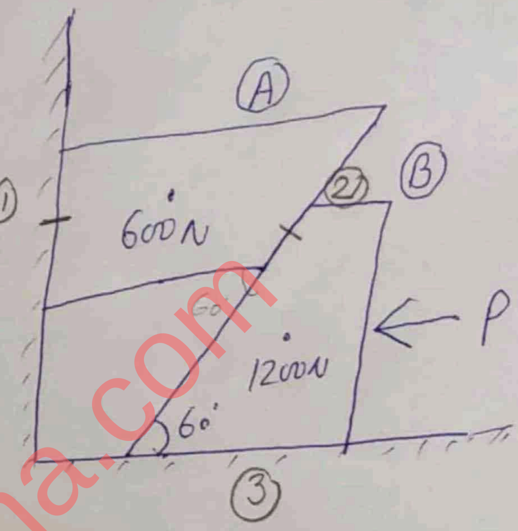
$R_1 = 549N$

$R_2 = 1214N$

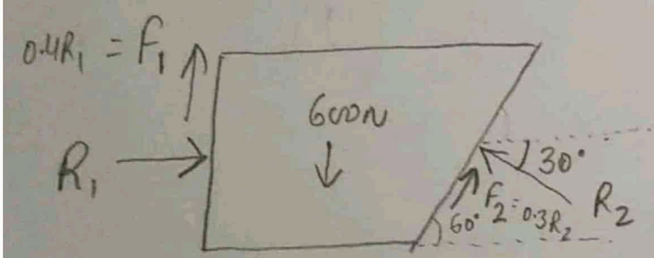
$R_3 = 1609.8N$

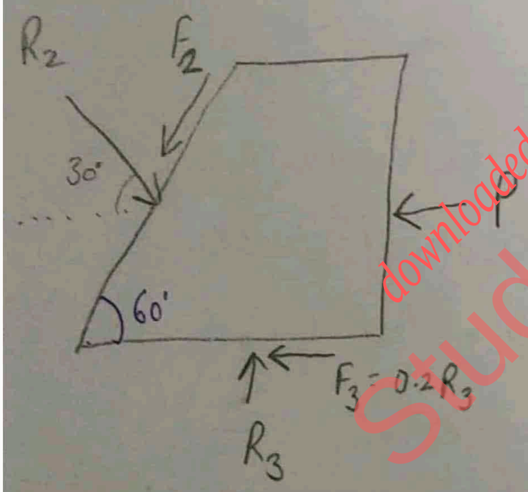


Q A system comprises two blocks A and B that against a wall and a floor as shown in fig. The weights of blocks are 600N and 1200N resp. Make calculations for the minimum horizontal force P that needs to be applied to keep the blocks in equilibrium. Assume the following values for $\mu$ : 0.2 for floor
0.3 for blocks
0.4 for wall.




Sol.



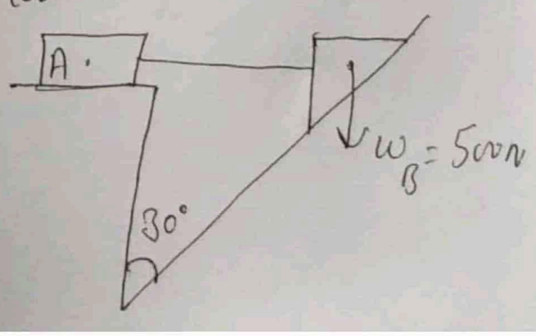

$R_2 = 573.4N$

$R_3 = 1635.67N$

$\boxed{P = 83.42N}$

---

Q Two blocks are connected by a horizontal link AB as shown in fig. What is the smallest weight $W_A$ of block A for which equilibrium can exist. Assume $\mu = 0.4$ for block A and floor.
$\phi = 20°$ for block B and plane.



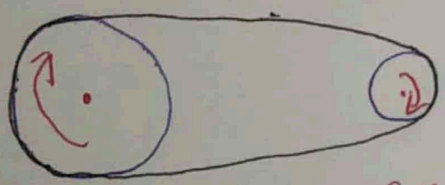

Ans
(1050N)

# Belt drive :

- Used for power transmission. (other method - ropes, chain, gear, clutch, shaft)
- It is a non positive drives (∵ of slip)

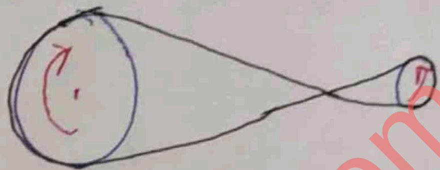
## Types of belt drive

i) open belt drive - rotation of driver and follower is in same direction.
ii) ~~closed~~ Crossed belt drive - " " " in opposite direction
iii) Compound belt drive - more than two pulleys are used.

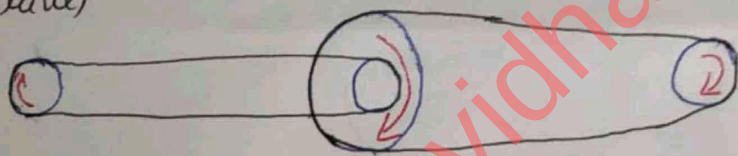


(Driver) (Follower)

(open belt drive)



( Cross belt drive)

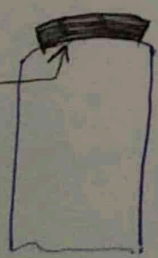


(Compound belt drive)

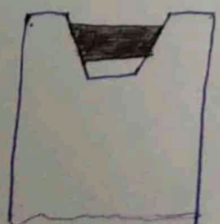


## Belt material and its types

~~Crowing~~ (crowning)



Flat belt



V-belt

Circular belt

Belt material - Rubber, leather, fabric, balata

Velocity ratio - It is the ratio of speed of driven pulley to that of driving pulley.

$d_1, d_2$ = dia of driver and driven pulleys

$\omega_1, \omega_2$ = angular velocities of driver and driven pulleys

$N_1, N_2$ = rotational speeds of driver and driven pulleys.

Assuming belt is inelastic and there is sufficient friction to prevent any slip, the pulleys will have the same linear speed.

$$\omega_1 \times \frac{d_1}{2} = \omega_2 \times \frac{d_2}{2}$$

$$\frac{\omega_2}{\omega_1} = \frac{d_1}{d_2} \quad \text{or} \quad \frac{2\pi N_2}{2\pi N_1} = \frac{d_1}{d_2}$$

$$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$



If thickness of belt ($t$) is taken into account,

$$\boxed{\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}}$$

If slip is also considered, then

$$\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}\left(1 - \frac{S}{100}\right) \qquad S = S_1 + S_2 + 0.01 S_1 S_2$$

$S$ = Total effective slip

$S_1$ = % slip b/w driver & belt

$S_2$ = % slip b/w belt & follower

Length of belt

$\Rightarrow$ open drive $\rightarrow$ $l = \pi(r_1+r_2) + \frac{(r_1-r_2)^2}{x} + 2x$

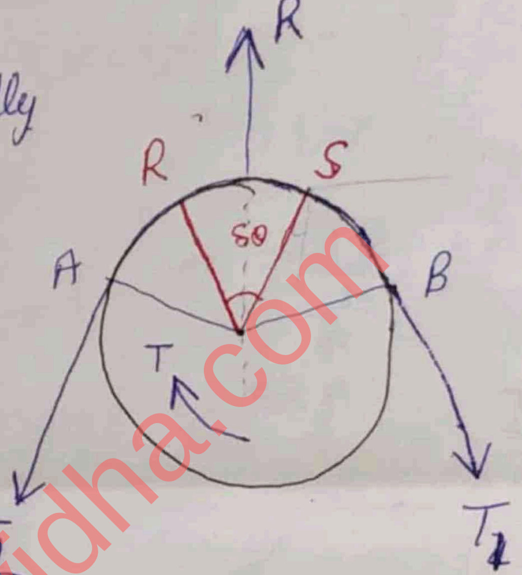
$\Rightarrow$ Crossed belt drive $\rightarrow$ $l = \pi(r_1+r_2) + \frac{(r_1+r_2)^2}{x} + 2x$

where $r_1, r_2$ radius of pulleys

$x$ = center distance centers of the two pulleys

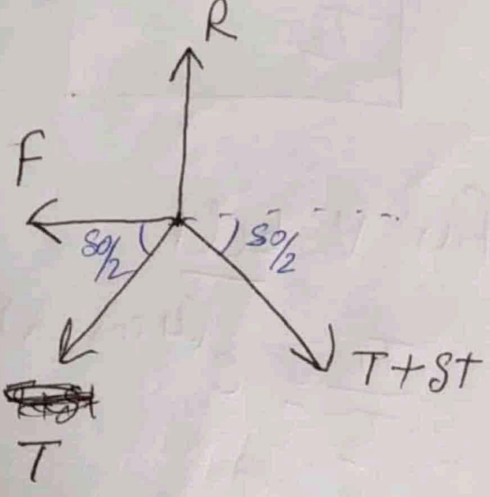
## Ratio of Tensions of driven pully

Consider the impending motion to be clockwise. $(T_1 > T_2)$




The angle subtended at the center of the pully by the position of belt in contact with it is called the angle of contact or the angle of lap $(\theta)$

Consider equilibrium of forces in the radial direction. (vertical)




$$R = (T+\delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

For small value of $\delta\theta$, $\sin\frac{\delta\theta}{2} \rightarrow \frac{\delta\theta}{2}$

$$R = (T+\delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2}$$

$$= T\,\delta\theta \quad -(1) \qquad \left[\text{neglect } \delta T\frac{\delta\theta}{2}\right]$$

Consider equilibrium of forces in tangential (horizontal) direction.

$$\mu R + T\cos\frac{\delta\theta}{2} = (T+\delta T)\cos\frac{\delta\theta}{2}$$

For small values of $\delta\theta$; $\cos\frac{\delta\theta}{2} \to 1$

$$\mu R = (T+\delta T) - T$$

$$\mu R = \delta T$$

$$R = \frac{\delta T}{\mu} \quad -(ii)$$

From (i) and (ii)

$$T\delta\theta = \frac{\delta T}{\mu}$$

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \int_0^{\theta} \mu\,\delta\theta$$

$$\log_e\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$



For V-belt

$$\frac{T_1}{T_2} = e^{(\mu\,\mathrm{cosec}\,\alpha)\theta}$$

$\theta$ = angle of lap

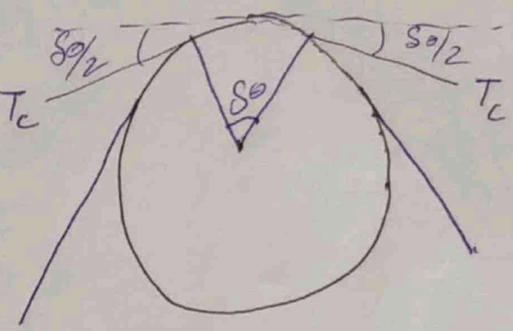
$2\alpha$ = angle of groove.

Centrifugal Tension : Due to the mass and speed, the belt is subjected to centrifugal force that acts radially outwards. This reduces [the normal r x n and hence frictional resistance.

Let $r$ = radius of pully

$V$ = speed of belt

$m$ = mass per meter length of belt




Then

length of elemental segment = $r\,\delta\theta$

mass of elemental segment = $m.r\,\delta\theta$

Centrifugal ~~Tension~~ Force $(F_c) = \frac{mv^2}{r} = \frac{mr\delta\theta V^2}{r} = m\delta\theta V^2$

This centrifugal force is counter balanced by tensions at the ends A and B of the elemental segment.

Considering equilibrium of forces in vertical direction

$$2T_c \sin\frac{\delta\theta}{2} = m\delta\theta V^2$$

$\sin\frac{\delta\theta}{2} \to \frac{\delta\theta}{2}$ [as $\frac{\delta\theta}{2}$ is very small)

$$2T_c\frac{\delta\theta}{2} = m\delta\theta V^2$$

$$\boxed{T_c = mV^2}$$

Tension on tight side ~~Side~~ = $T_1 + T_c$

" " slack " $= T_2 + T_c$

Power Transmission -

$T_1$ = Tension on tight side

$T_2$ = Tension on slack side

$V$ = Velocity of the belt

Then, effective turning force = $T_1 - T_2$

~~Work done = $(T_1 - T_2) V$ Nm/s~~

Power = $(T_1 - T_2) V$ watts

Second method

Turning moment acting on the pulley = $(T_1 - T_2) r$

work done per second = $(T_1 - T_2) r\omega$

$= (T_1 - T_2) V$

Condition for transmission of max. power

$$P = (T_1 - T_2) V$$

$$= T_1 \left[1 - \frac{T_2}{T_1}\right] V = T_1 \left(1 - \frac{1}{e^{\mu\theta}}\right) V$$

$$= (T - T_c) KV \quad \left[\begin{array}{l} T = \text{max. permissible tension} = T_1 + T_c \\ \left(K = 1 - \frac{1}{e^{\mu\theta}}\right) \end{array}\right]$$

$$= (T - mv^2) KV$$

$$= (TV - mV^3) K$$

$$\frac{dP}{dV} = T - 3mV^2 = 0$$

$$T = 3mv^2 = 3T_c$$

$$\boxed{V = \sqrt{\frac{T}{3m}}} \qquad \boxed{T_c = \frac{T}{3}}$$

Centrifugal Tension

$$\boxed{T_c = mV^2}$$

[$m$ = mass per meter length of belt

$V$ = speed of the belt.]

Total tension on tight side = $T_1 + T_c$

" " " slack " = $T_2 + T_c$

## Initial tension ($T_0$)

During motion and power transmission,
The tight side of the belt stretches until the tension increases from $T_0$ to $T_1$.

The corresponding increase in length of belt on the tight side is $= \alpha (T_1 - T_0)$ —(i)

where $\alpha$ = coeffient of belt length per unit force.

The resulting decrease in length of belt on slack side $= \alpha (T_0 - T_2)$ —(ii)

∵ The belt is inelastic, then length of belt remains unchanged

$$\alpha (T_1 - T_0) = \alpha (T_0 - T_2)$$

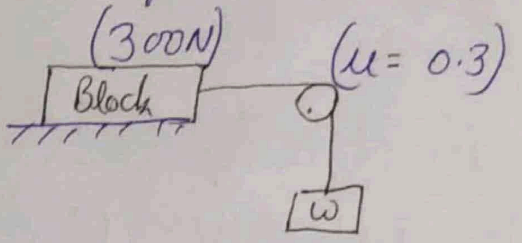
$$\boxed{T_0 = \frac{T_1 + T_2}{2}}$$

If centrifugal tension is taken into account.

$$\boxed{T_0 = \frac{T_1 + T_2 + 2T_c}{2}}$$

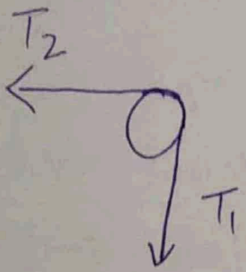
---

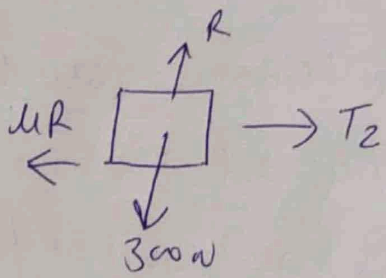
Q Determine the minimum value of weight W required to cause motion of the block on surface as shown in fig. Take ($\mu = 0.6$)




Angle of lap = 90°

Sol.




$T_1 = W$

$\frac{T_1}{T_2} = e^{\mu\theta}, \quad 1.60$

$T_2 = \mu R$

$R = 300N$

$T_2 = 180N$

$T_1 = 288N$

Lifting machine: A machine may be defined as a device that receives energy in some available form and uses it for doing a particular useful work.

Some basic machines are

i) Lever    ii) Pulley    iii) Inclined plane

iv) Screw    v) Wedge    vi) wheel and axel

The machine which are used to lift heavy loads are called lifting machines.

Basic Definitions:

⇒ Mechanical Advantage (MA) $= \frac{W}{P} = \frac{\text{weight lifted}}{\text{Effort applied}}$

⇒ Velocity Ratio (VR) $= \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{y}{x}$

⇒ Input work - $P.y$

⇒ Output work - $W.x$

⇒ Efficiency of machine -

$$\eta = \frac{\text{Useful work done by the machine}}{\text{work expended on the machine}}$$

$$= \frac{\text{Output of the machine}}{\text{Input of the machine}}$$

$$= \frac{Wx}{Py} \Rightarrow \frac{W}{P} \times \frac{1}{\frac{y}{x}}$$

$$\boxed{\eta = \frac{MA}{VR}}$$

[In a simple machine, a small force when applied through a large distance overcomes a large force through a small distance.]

Reversible and Irreversible machine (Self locking)

| Reversible | Irreversible |
|---|---|
| Load fall (Pulley) | Load does not fall. (Screw Jack) |

In an irreversible machine, some work done is lost due to friction

Friction work = Input - Output = $P.y - Wx$

On removal of effort, the load will not fall if the friction work is more than the output of machine.

$$(Py - Wx) > Wx$$

$$Py > 2Wx$$

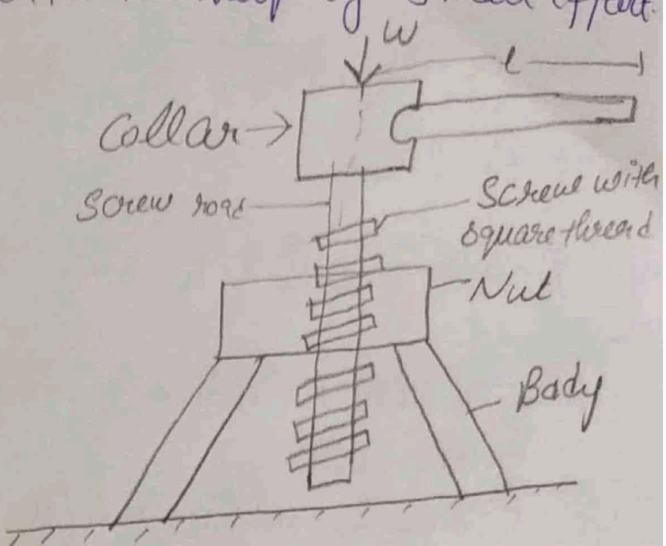
$$\frac{Wx}{Py} < \frac{1}{2}$$

$$\eta < \frac{1}{2}$$

$$\eta < 50\%$$

$\therefore$ For irreversibility or self-locking of a machine the $\eta < 50\%$

Screw Jack - It is a simple machine used for lifting heavy loads, through short distances, with the help of small effort.




when one rotation is given to the handle,

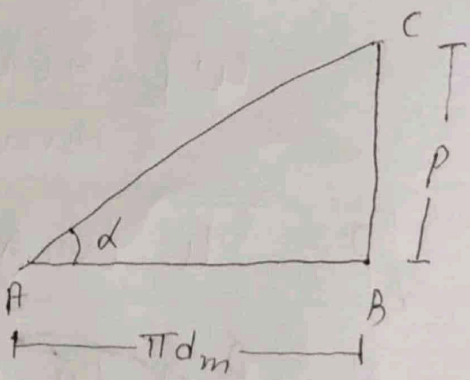
distance moved by effort = $2\pi l$

distance through which load is lifted is pitch of the thread ($p$)

$$V.R. = \frac{2\pi l}{p}$$

The working principle of a screw jack is the same as that of the inclined plane.

This fig shows development of one complete turn of a screw thread. Distance AB will be equal to the circumference and distance BC is equal to the pitch of the screw.



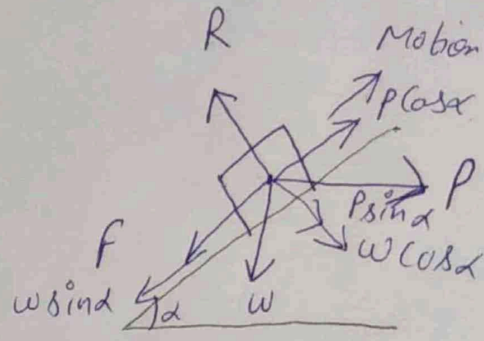

$$\tan\alpha = \frac{p}{\pi d_m}$$

$$\left[ \begin{array}{l} \alpha : \text{helix angle} \\ d_m : \text{mean dia. of thread} \end{array} \right]$$

Effort required to lift the load:

Consider equilibrium condition:

along the plane




$$P\cos\alpha = \mu R + W\sin\alpha \quad \text{- (i)}$$

$$R = W\cos\alpha + P\sin\alpha \quad \text{- (ii)}$$

From (i) & (ii)

$$\therefore \quad P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha)$$

$$P(\cos\alpha - \mu\sin\alpha) = W(\sin\alpha + \mu\cos\alpha)$$

$$P = \frac{W(\sin\alpha + \mu\cos\alpha)}{\cos\alpha - \mu\sin\alpha}$$

$$\text{But } \because \mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$$

$$P = \frac{W[\sin\alpha\cos\phi + \mu\sin\phi\cos\alpha]}{\cos\alpha\cos\phi - \sin\phi\sin\alpha}$$

$$= \frac{W[\sin(\alpha+\phi)]}{\cos(\alpha+\phi)}$$

$$\boxed{P = W\tan(\alpha+\phi)}$$

Here P is the effort applied at the mean radius of screw jack. But in practice, the effort is applied at the end of the handle of the jack.

Let $P_h$ = horizontal force applied at the end of handle.

$l$ = length of handle.

$$P_h \times l = P \times \frac{d_m}{2}$$

$$= W \tan(\alpha+\phi) \frac{d_m}{2}$$

$$\boxed{P_h = \frac{W d_m}{2l} \tan(\alpha+\phi)}$$

$$P = W \tan(\alpha+\phi)$$

In the absence of friction $\phi = 0$ [Ideal condition]

$$P_o = W \tan\alpha$$

$$\eta = \text{Efficiency} = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{W \tan\alpha}{W \tan(\alpha+\phi)} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

$$\boxed{\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}}$$

To find max efficiency, differenciate $\eta$ with respect to $\alpha$

$$\frac{d\eta}{d\alpha} = \frac{d}{d\alpha}\left[\frac{\tan\alpha}{\tan(\alpha+\phi)}\right] = 0$$

$$\frac{\sec^2\alpha \tan(\alpha+\phi) - \sec^2(\alpha+\phi)\tan\alpha}{\tan^2(\alpha+\phi)} = 0$$

$$\sec^2\alpha \tan(\alpha+\phi) = \sec^2(\alpha+\phi)\tan\alpha$$

$$\frac{1}{\cos^2\alpha} \times \frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)} = \frac{1}{\cos^2(\alpha+\phi)} \times \frac{\sin\alpha}{\cos\alpha}$$

$$2\cdot\sin(\alpha+\phi)\cos(\alpha+\phi) = 2\cdot\sin\alpha\cos\alpha$$

$$\sin 2(\alpha+\phi) = \sin 2\alpha = \sin(\pi-2\alpha)$$

$$2(\alpha+\phi) = \pi - 2\alpha$$

$$\alpha = \frac{\pi}{4} - \frac{\phi}{2} = 45^\circ - \frac{\phi}{2}$$

$$\boxed{\alpha = 45^\circ - \frac{\phi}{2}}$$

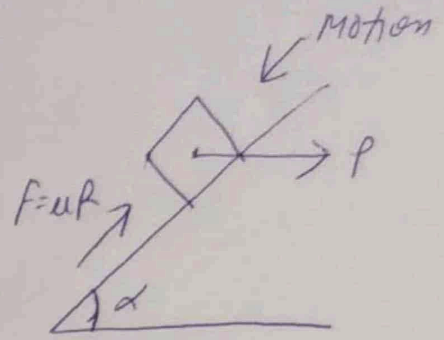
$$\eta_{max} = \frac{\tan\left[45-\frac{\phi}{2}\right]}{\tan\left[45-\frac{\phi}{2}+\phi\right]} = \frac{\dfrac{1-\tan\frac{\phi}{2}}{1+\tan\phi/2}}{\dfrac{1+\tan\phi/2}{1-\tan\phi/2}} = \left(\frac{1-\tan\frac{\phi}{2}}{1+\tan\frac{\phi}{2}}\right)^2$$

$$= \left[\frac{\cos\frac{\phi}{2}-\sin\phi/2}{\cos\frac{\phi}{2}+\sin\frac{\phi}{2}}\right]^2 = \frac{1-2\cos\frac{\phi}{2}\sin\frac{\phi}{2}}{1+2\cos\frac{\phi}{2}\sin\frac{\phi}{2}} = \frac{1-\sin\phi}{1+\sin\phi}$$

$$\boxed{\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}}$$

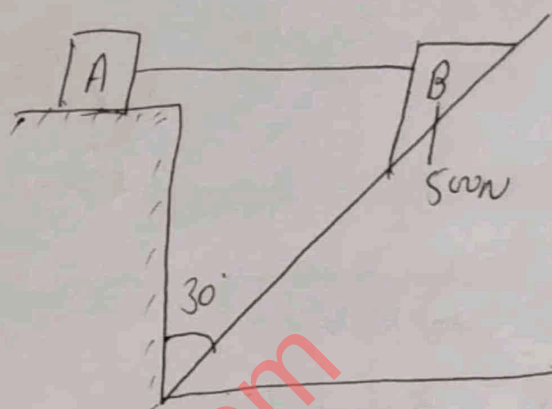
Lowering of load:

$$P = w \tan(\alpha - \phi)$$



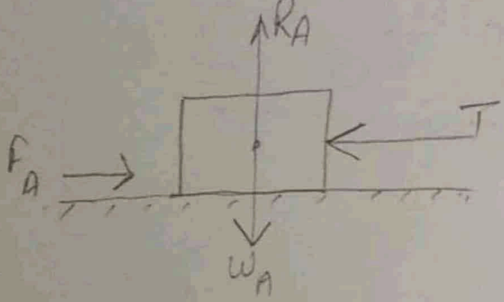
6.6 (Friction)

Q Two blocks are connected by a horizontal link AB and rest on two planes as shown in fig. what the smallest weight $W_A$ of the block A for which equilibrium can exist? Assume $\mu$ for the block A and horizontal surface to be 0.4 and the angle of friction for the block B on the inclined plane is $\phi = 20°$. [1050N]



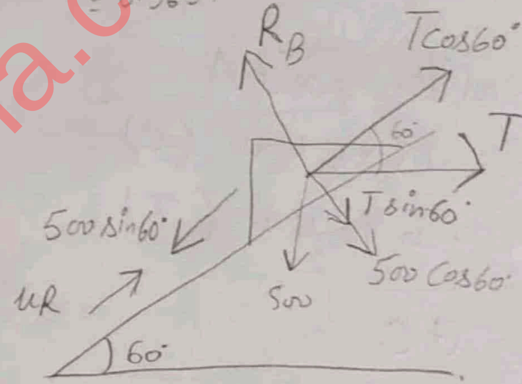

$\mu = \tan 20° = 0.3639$

Ans




$T = F_A$ (i)

$R_A = W_A$ (ii)




$R_B = T\sin 60 + 500\cos 60°$ — (iii)

$T\cos 60° + \mu R_B = 500\sin 60°$ -(iv)

$T\cos 60° + \mu[T\sin 60 + 500\cos 60°] = 500\sin 60$

$T[\cos 60 + \mu \sin 60°] = 500\sin 60 - \mu\, 500\cos 60°$

$T = 419.549N$

From (i)

$T = F_A = 419.549 = \mu R_A$

$R_A = 1048 N$

$W_A = R_A = 1048 N$